ON DEFORMABILITY OF SOILS IN A COMPLEX STRESS STATE

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Possibilities of using the Hencky-Nadai theory for complex loading of soils are experimentally investigated. It is found that for sand and loam of disturbed structure the necessary premises of the Hencky-Nadai theory are fulfilled under complex loading on fairly smooth loading paths.

The modern methods of calculation in soil mechanics are based on the simplest deformation theory of Hencky-Nadai

$$S_{ij} = G_s \varepsilon_{ij} \tag{1}$$

where S_{ij} is the stress deviator, ε_{ij} is the strain deviator, and G_S is secant modulus of the curve of stress intensity depending on strain intensity.

The observation of the initial premises of this theory [1] (the coincidence of the principal axes of stress and strain tensors and the similarity of their deviators) has been investigated for complex loading of soils, including the case where the directions of the principal stress axes change. Such a formulation of the investigation is determined by the fact that the loading of an element of soil, as a rule, is complex. At the same time experimental studies of complex loading of soils nowadays are almost nonexistent.

The experiments have been formulated on hollow cylindrical testpieces of soil loaded by an axial force and hydraulic pressure which was not the same inside and outside the testpieces. Various combinations of these loads allow the deformation of soil to be investigated both under the conditions of simple loading and under those of complex loading. An application of a twisting moment to the ends leads to the position of the axes of the principal stresses σ_1 and σ_3 being altered.

The experimental installation allowed testpieces of height 80 mm, with the internal and external diameters 35 and 60 mm respectively, to be tested. The application of the vertical load and the twisting moment was effected by mechanical systems, while the application of the hydrostatic pressure was effected by compressed air through glycerine and rubber shells. The vertical deformation of the testpiece was measured by an indicator of the clock gage type connected with the loading rod, with an accuracy of 0.01 mm. The radial and tangential deformations were determined from the changes in the outer and inner diameters of the testpiece, measured from the volume of glycerine in the inner and outer cavities of the instrument with an accuracy up to 0.05 cm^3. The angular deformation was determined by a geodesic limb fixed to the loading rod, with an accuracy of 0°02'. The loads specified were fixed by standard manometers and a dynamometer with a scale divided into 0.05 kg/cm^2.

The soils investigated are characterized by the following indices. The sand is of medium grain, homogeneous, of medium density with a specific weight of 2.65 g/cm^3; the porocity coefficient is 0.57, and the predominant fraction of the grain content is 0.25 to 0.5 mm.

The loam of disturbed structure has a moisture content 10 at the yield point, and a weight content of moisture of 12.3%. Its porosity coefficient is 0.76, the degree of water saturation is 0.44, and the specific weight is 2.72 g/cm^3.

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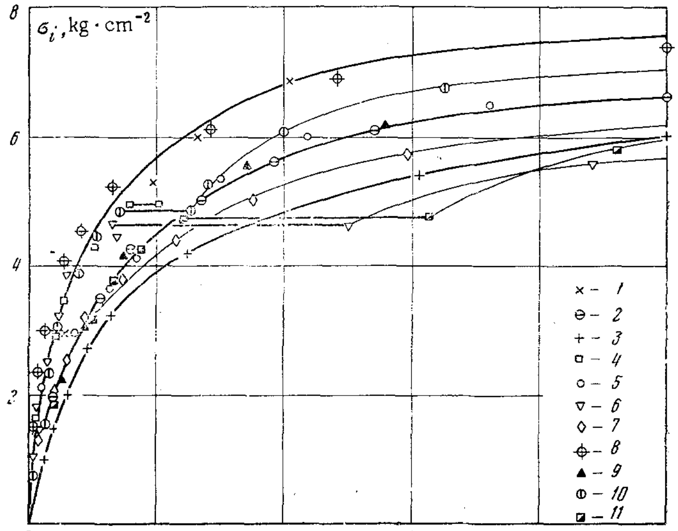

Fig. 1

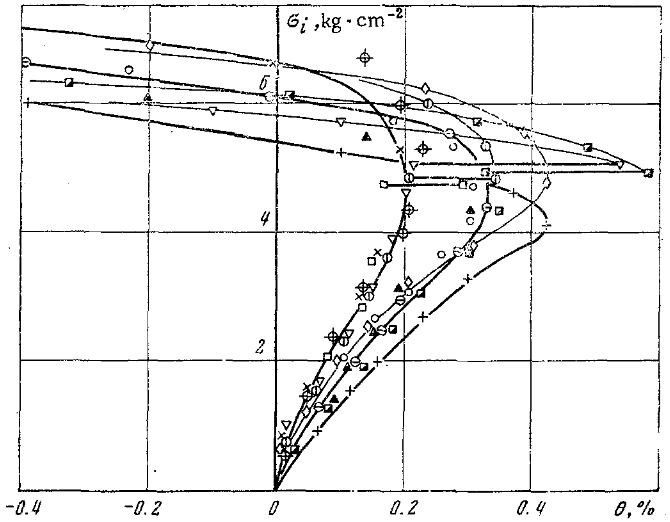

Fig. 2

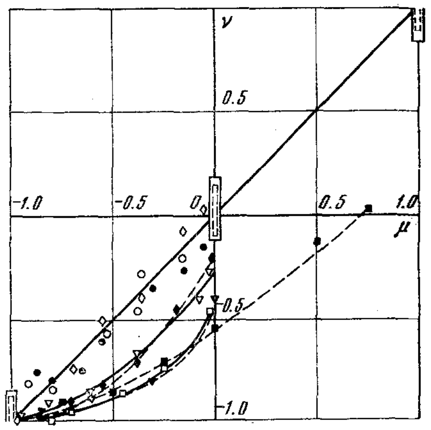

Fig. 3

The initial isotropy of the properties of the soils is experimentally confirmed by means of an identical program from testpieces oriented in a different manner.

The loading of the soils was carried out by steps with strain stabilization after each load step.

All experiments were set up using a magnitude of the average pressure σ which remained constant during the course of loading, so that only the components of the stress deviator increased. The value of σ is taken equal to 5 kg/cm^2.

The experiments carried out can be divided into three groups:

1) experiments of simple loading during which the components of the stress deviator increased proportionally and the parameter of the form of the stress state (parameter of Lode)

$$\mu = \frac{2\sigma_2 - \sigma_1 - \sigma_3}{\sigma_1 - \sigma_3}$$

remained constant;

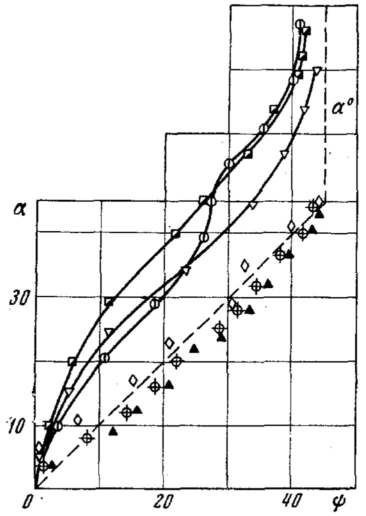

Fig. 4

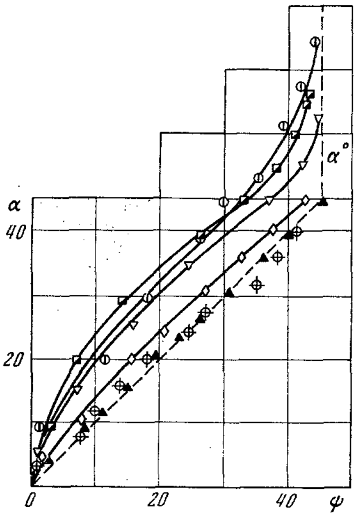

Fig. 5

2) experiments of complex loading (μ is variable) with a fixed position of the axes of principal stresses;

3) experiments of complex loading during which the direction of the axes of principal stresses was varied.

The simple loading has been carried out both for sand and for loam, for three values $\mu = -1$, 0, and +1 (the tests* 1, 2, and 3). The experiments of the second group include two tests: 4 and 5. In the test 4 on the first stage the loading is simple ($\mu = -1$); then for a constant value of the stress intensity σ_i (5.1 kg/cm^2 for loam and 4.9 kg/cm^2 for sand) the parameter of the form of the stress state was varied in steps from −1 to 0. In an analogous test with sand the value of μ increased between the limits from −1 to +0.75.

The test 5 differed from the test 4 by the fact that the same stress state ($\mu = 0$, $\sigma_i = 5.1$ kg/cm^2 for loam and 4.9 kg/cm^2 for sand) was reached by a simultaneous increase in σ_i and a variation in μ from −1 to 0. The concluding stage of the tests 4 and 5 consisted of increasing σ_i right up to the failure with $\mu = 0$.

Complex loading with variation of the position of the stress principal axes has been carried out in the tests 6, 7, 8, 9, 10, 11.

The program of variation of the principal stresses, as well as σ_i and μ in the tests 6 and 7, completely coincides with the program of the tests 4 and 5 respectively. However, simultaneously with the variation of the form of the stress state the axes of the principal stresses were rotated through 45°.

In the tests 8 and 9 the loading took place with a constant value of the Lode parameter equal to −1 and 0. The increase in the stress intensity was accompanied by a rotation of the principal stress axes through 45°.

In the tests 10 and 11 the rotation of the axes of principal stresses through 45° was effected after preliminary simple loading ($\mu = -1$ and 0). In the process of rotation the values of the principal stresses were maintained constant. The subsequent loading has been carried out for a constant value of μ.

The results of the tests† 1-11 represented in Figs. 1-5 allow us to estimate the degree of violation of the premises of the simplest deformation theory under the conditions of complex loading: the coincidence of the axes of the stress and strain tensors, the similarity of their deviators. Side by side with this we consider the violation of the relationships between the stress and strain invariants which are characteristic for simple loading.

The dependence of the strain intensity e_i and the volume strain θ on the stress intensity σ_i, for the variety of sand investigated, is shown in Fig. 1 and Fig. 2.

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*Since the investigation programs of sand and loam are analogous, the numeration of tests here is the same.
†The conditional test symbols are presented in Fig. 1.

In Fig. 3 we have presented the relationships between μ and the analogous parameter for strain

$$\nu = \frac{(2e_2 - e_1 - e_3)}{(e_1 - e_3)}$$

The experimental points obtained with simple loading lie within the zones bounded by the rectangles. Dashed lines and blackened dots refer to tests on sand, while continuous lines and hollow circles refer to tests on loam.

The variation of the angle of rotation of the axes of principal strains

$$\psi = \frac{1}{2} \operatorname{arc\,tg} \frac{e_{13}}{e_{11} - e_{33}}$$

for rotation of the axes of principal stresses through the angle

$$\alpha = \frac{1}{2} \operatorname{arc\,tg} \frac{2\sigma_{13}}{\sigma_{11} - \sigma_{33}}$$

is shown in Fig. 4 for loam and in Fig. 5 for sand.

On the same graphs we have shown the variation of ψ in the process of the subsequent stage of loading in the tests 6, 10, 11 ($\alpha^{\circ} = 45° = \text{const}$ and $\mu = \text{const}$).

Consideration of the data presented in Figs. 1-5 allows us to draw the following conclusions.

Under simple loading for each value of μ we have obtained its graphs showing the dependence of σ_i on e_i and σ_i on θ (Fig. 1, Fig. 2). Thus the relationships between strain and stress invariants must be written in the form

$$e = e(\sigma, \sigma_i, \mu), \quad e_i = e_i(\sigma, \sigma_i, \mu) \tag{2}$$

The necessity of introducing the mean pressure into Eq. (2) follows from the consideration of numerous experimental data [2, 3].

The parameters μ and ν under simple loading can be taken as equal (Fig. 3), i.e., the deviators of stresses and strains are similar. Consequently, for sand, just as for loam, the Hencky relationships are true under simple loading. This result has experimentally been obtained in other works [3].

Experiments showed that the laws of simple loading are retained also for certain trajectories of complex loading. Thus, in tests where variation of the form of stress state and rotation of the principal axes of stresses were accompanied by an increase in the stress intensity (the tests 5, 7, 8, 9) either no deviations from the relationships of simple loading were obtained, or such deviations were small. In the tests 5, 7 and 2, for $\mu = 0$, $\sigma_i = 5.1$ kg/cm^2 for loam and 4.9 kg/cm^2 for sand, and independently of the presence of rotation of the axes of principle stresses, the quantities e_i and θ are close to one another for the different tests considered, and only slightly depend on the trajectory of the subsequent loading, i.e., the relationships (2) are fulfilled.

It is of considerable interest to compare the results thus obtained with the data of other investigations. Experiments with metals [4] also show that complex loading accompanied by an increase in σ_i does not lead to a significant violation of the relationships of the deformation theory of plasticity. For metals the deviations from the relationships of simple loading is small also in the case where rotation of the axes of principal stresses is carried out for an unaltered form of the stress state simultaneously with an increase in σ_i [5]. The tests 8 and 9 carried out by the authors confirm this result also for soils. The graphs of these tests (Figs. 1 and 2) practically coincide with the graphs of the tests 1 and 2 which have been obtained under the conditions of simple loading.

In the experiments 5, 7, 8, 9 considered here, the equation $\mu = \nu$ (Fig. 3) is satisfied with a sufficient degree of accuracy, and $\alpha = \psi$ (Fig. 4 and Fig. 5), i.e., the principal axes of the stress and strain tensors coincide, and their deviators are similar. Thus, the premises of applicability of the Hencky equations (1) are satisfied. Certain deviations from the relationships of simple loading have been obtained for sand in the test 7, in which loading took place with a simultaneous increase in σ_i, variation of μ from -1 to 0, and rotation of the principal axes of stress through 45°. Somewhat larger strains e_i and θ and somewhat smaller angles ψ, in comparison with simple loading, have been obtained.

Completely different results have been obtained under complex loading which was preceded by a stage of simple loading (the tests 4, 6, 10 and 11). A subsequent variation of μ or α (or μ and α simultaneously) gave rise to sharp deviations from the relationships of simple loading. An increase of the Lode parameter from -1 to 0 (the test 4) with an unaltered value of the stress intensity leads to an increase in e_i and θ (Fig. 1 and Fig. 2) which, however, is less than the difference between the lines of simple loading for $\mu = -1$ and 0, respectively. The same result is obtained in an analogous test with loam.

A variation in the orientation of the principal stress axes, for a constant σ_i, leads to a considerable increase in the strains e_i and θ (the tests 10 and 11). Thus, for trajectories of the type under consideration, for which the presence of a portion of simple loading preceding complex loading is characteristic, Eqs. (2) are unsuitable, since they do not take into account the influence of the trajectory.

Analogous experiments with metals show that complex loading following simple loading also leads to a violation of the relationships of simple loading. These violations are the severest when σ_i remains constant [4, 6].

Side by side with a deviation from Eqs. (2) in the tests 4, 6, 10 and 11 we have a considerable violation of the co-axiality of the stress and strain tensors, as well as the similarity of their deviators (Figs. 3, 4, 5). The conditions of applicability of Hencky equations are grossly violated.

The investigations of complex loading of two different soils carried out here enabled us to establish certain general features of deformation. It is shown that for certain trajectories (and, apparently, for certain soils and their physical states) the relationships of simple loading are retained under complex loading. These trajectories are characterized by the absence of a sharp change in the form of the stress state and orientation of the principal stress axes, while the loading takes place with a growing intensity of the stresses. Thus, the description of processes of complex loading along trajectories of this type can be effected by equations of the simplest deformation theory of Hencky-Nadai.

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